Discrete energy eigenstates of neutrons in a linear gravity potential:

A simpler (and more complicated) means of calculating the acceleration due to gravity

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Introduction:

In this paper, I describe and propose a new method for measuring acceleration due to gravity using neutrons in a linear gravity potential. Through the merging of the theory of general relativity, classical mechanics, and quantum mechanics, low-energy neutrons can be used to calculate acceleration due to gravity by using their eigenvalues (i.e. specific values) of their eigenstates (i.e. positions of each energy state) to determine the probability of their location in a linear gravity potential.

It is important to understand the foundations of the theories of general relativity and quantum mechanics to understand the mechanism by which this method functions. Gravity measuring methods used most often today assume gravity to be a force field in which two objects are attracted across a distance in a three-dimensional vector field in accordance with classical Newtonian gravity. The force of this attraction is given by equation 1, with G being the gravitational constant, m_1 and m_2 being the mass of each object, and r being the distance between the two objects.

Equation 1

$$F_g = \frac{Gm_1m_2}{r^2}$$

On large scales, classical Newtonian gravity is incredibly accurate. However, on very small scales, such as on the scale of a neutron (roughly three femtometers, or 3E-15 meters, radius) gravity is not gradual but discretized. Neutrons, unlike Newton's apple, fall in steps, not smoothly. The answer as to why lies in quantum mechanics, in the wave-particle duality of matter and the Uncertainty Principle, as well as the theory of general relativity.

The theory of general relativity notes that space and time are but one continuum, linked and inseparable. On the non-quantum scale, gravity is a manifestation of this space-time fabric "pushing" objects that displace it to their "lowest potential." It can be visualized like a trampoline. If an object is placed on the trampoline, the plane distorts to a lowest point centering around that object. If another object is added to the trampoline, in this curved state it will move towards the previous object, or the object move towards it, depending on which displaces the trampoline fabric more based on which is more massive. Therefore, gravity isn't an attraction between two objects but space taking a shape based on mass and mass moving according to that shape. Space-time and gravity act in this way, not as a force, "Gravity isn't a force, it's simply space pushing on the two masses" (Dr. Michio Kaku) For non-quantum massive objects, they observe the curvature of spacetime as a perfectly smooth plane, much like a bowl. Quantum objects, like neutrons, however, view space-time discretely, much like a stadium with steps. These particles can only take up discrete portions of gravitational energy due to the limited curvature of space-time. It can be visualized thinking about the stadium example again. If a large stadium has small steps from the top to the bottom, a small ball moving from the top of the stadium towards the center will fall discretely. If a probability function for the height of such a ball were graphed, it would show the greatest probability of the ball's location to be at each step, with less probability of the ball being between steps during free fall. A large ball, however, if it is significantly larger than the steps in the stadium, will treat the steps more like a plane, and its probability function displaying more of a normalized curve than a wave. The reason why neutrons treat gravity and space discretely and why non-quantum particles do not is determined by this understanding of space as well as their respective wave-functions.

Matter acts not only as discrete particles but also as waves. Thomas Young, in the double slit experiment, showed that light, when passed through two slits, diffracts, thereby showing its wave-form. Plane waves of the light diffract when passed through the slits, curving the lights waves causing a pattern due to the interference of the waves. This dual slit experiment was recreated using electrons, and it showed that the electron not only passed through both slits but caused an interference pattern just like the light waves. Erwin Schrödinger developed the equations necessary to calculate the wave functions of particles in a linear potential. On non-quantum length-scales, objects' wave functions exhibit Compton wavelengths (Planck constant divided by the product of the mass and the speed of light) smaller than that of Planck time, meaning they, like the large ball, do not see space as discretized. The proposed method for measuring gravitational acceleration described in this paper uses this theoretical framework of gravity and wave function state of matter.

Quantum gravity meter:

The instrumentation used in this method has been used on numerous occasions to describe eigenstates of neutrons in a linear gravity potential, mapping their respective wave functions (e.g V.V.Nesvizhevsky et. al., 2002; Y. Kamiya et. al., 2014; Harmut Abele, 2012.) Many of these particle physicists also had the goal of observing and describing higher-dimensional gravity-like forces acting on the neutrons such as axion interaction or hypothetical Yukawa-type forces. None of them, however, developed the method as a means of measuring gravitational acceleration, despite its ability to be used for such a purpose.

The setup consists of a neutron emitter that emits a beam between two mirrors placed horizontally, "bouncing" the neutrons off the lower mirror and into a neutron detector (e.g. a CR39-plastic detector.) The bottom mirror repels neutrons, the top mirror is a neutron scatter, or rough mirror [Figure 1.] The height above the bottom mirror the neutron reaches is gravity (or Newtonian potential) dependent. Much like a bouncing ball, the probability of the neutron's height above the reflecting surface approaches zero as height decreases [Figure 2.] Ultra cold neutrons are used due to their low energy caused by their slower motion. Most experiments used reactor neutrons, which are colder than 1 mK. The higher the energy state of the neutron, the more classical the motion and the probability density curve [Figure 3.] The relative heights above the mirror of each eigenstate of the neutrons as described by Schrödinger equation and solved by Airy functions can be used to determine the gravitational acceleration.

Numerical methods:

The necessary variables in calculating a particle's eigenfunction are *m* the mass of the particle, *z* the height above the mirror, and *g* the gravitational acceleration. The only constant used is ħ the reduced Planck constant (6.58211928×10−16 eV*s.) Their eigenfunctions are described by Schrödinger equation (equation 2.)

Equation 2

$$\left\{-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}z^2} + V(z)\right\}\psi_n(z) = E_n\psi_n(z)$$

Where $V(z)$ is the linear potential described by $V(z) = mgz$ and ψ_n is the eigenfunction at a particular eigenstate "*n*". E_n is the eigenenergy at a particular eigenstate "*n*." From the Schrödinger equation we can obtain its dimensionless form, equation 3.

Equation 3

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}\xi_n^2} - \xi_n\right)\psi_n(\xi_n) = 0,$$

Here, the scalar ξ_n is equal to $z/z_0 - E_n/E_0$ where z_0 and E_0 equal equation 4 and 5, respectively.

Equation 4

$$z_0 = \left(\frac{\hbar^2}{2m^2 g}\right)^{1/3}$$

Equation 5

$$E_0 = \left(\frac{mg^2\hbar^2}{2}\right)^{1/3}$$

The solutions to the dimensionless form of the Schrödinger equation are described by Airy special functions $\mathrm{Ai}(\xi_n)$ and $\mathrm{Bi}(\xi_n)$. The resulting probability wave functions can be seen in figure 4. Or, if we would like to graph z_n with E_n, figure 5.

These wave functions would shift as gravitational acceleration increases or decreases. The system is incredibly sensitive, with a microgal change in acceleration yielding nearly two-fold scalar changes to the wave functions. This poses a challenge to those trying to use the system to measure linear gravity acceleration as most anomalies of interest are significantly greater than the microgal scale. One option is to use the system strictly for monitoring purposes, noting minute changes in gravity for a single location rather than changes in gravity over a distance. However, through increasing the height of the rough mirror we can decrease the eigenenergies of the neutrons by increasing the height of the neutron scatterer and decreasing the length of the mirror.

Limitations:

Due to the ultra-sensitive nature of the method, coupled with its high cost and limited mobility due to relying on cold neutrons, the method isn't practical for exploration purposes. Monitoring stations could be a practical use for the method, or if a single area were of interest numerous stations using the method could be used to provide gravitational acceleration measurements with unprecedented accuracy. Most experiments carried out using the method lasted about two days per measurement, but it should be noted that most of those experiments were designed to look for near infinitesimal gravity-like forces that are observed though deformations of the wave functions of the neutrons. If the method were used simply to obtain an accurate gravitational acceleration measurement it would not require as long of a run time to obtain viable results.

In addition, the cost for the current method is high. Neutron emitters are extremely expensive, though recent developments in the technology have greatly decreased their cost. Neutrons also must be extremely cold, either requiring their source to be from a reactor or an energy-intensive shielding process be used to lower their temperature before they pass between the mirrors. Neutron detectors are also expensive, and are only accurate down to about 1.5 μ meters. Neutron detectors also only last so long as the method is destructive.

Conclusion:

While the methods may be expensive and hypersensitive, using neutrons in a linear gravity potential to determine gravitational acceleration currently could have numerous uses such as monitoring or long-term studies of a localized area. Future developments in neutron emitter technology or neutron detector technology could increase the viability of the method for exploration purposes.

Figure 1

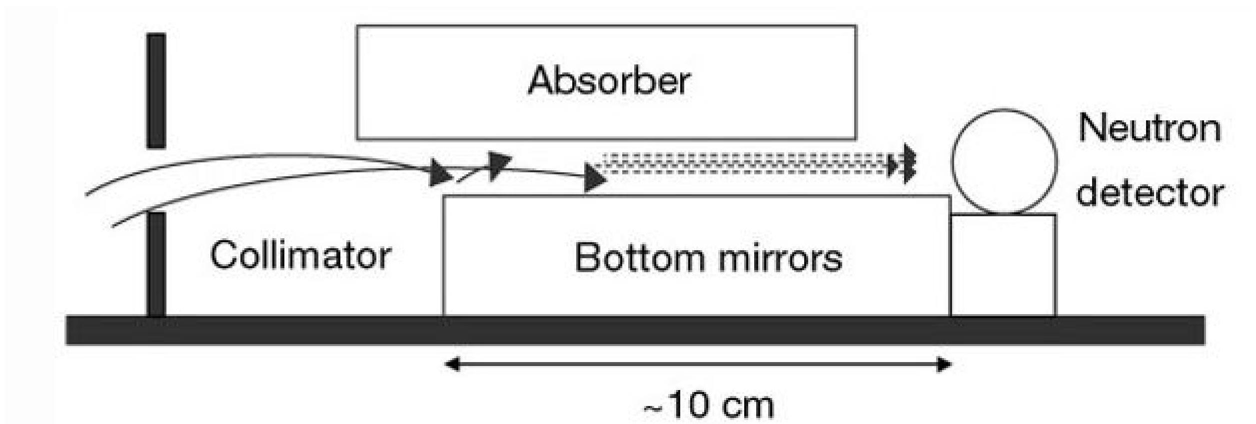

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Figure 2

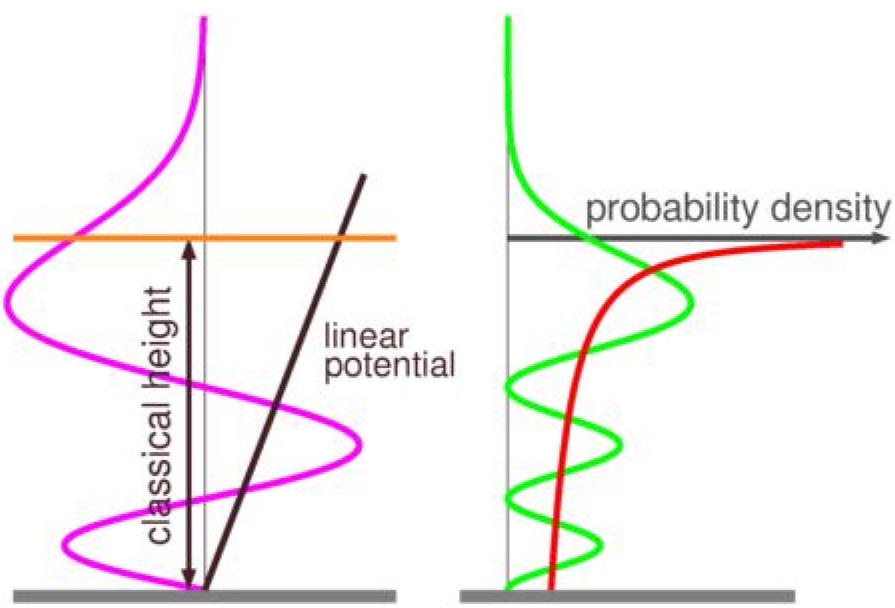

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Figure 3, neuron eigenfunction at 60th energy state.

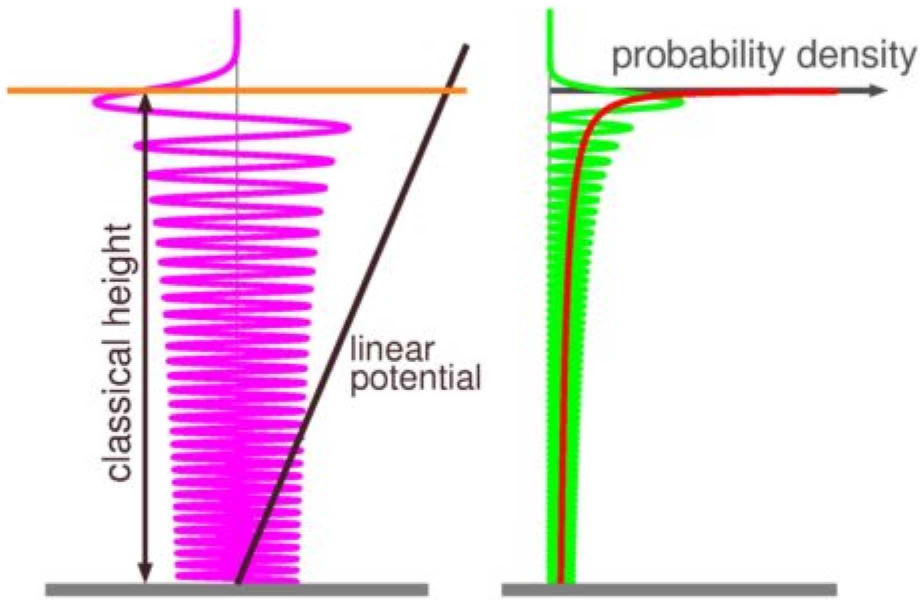

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Figure 4.

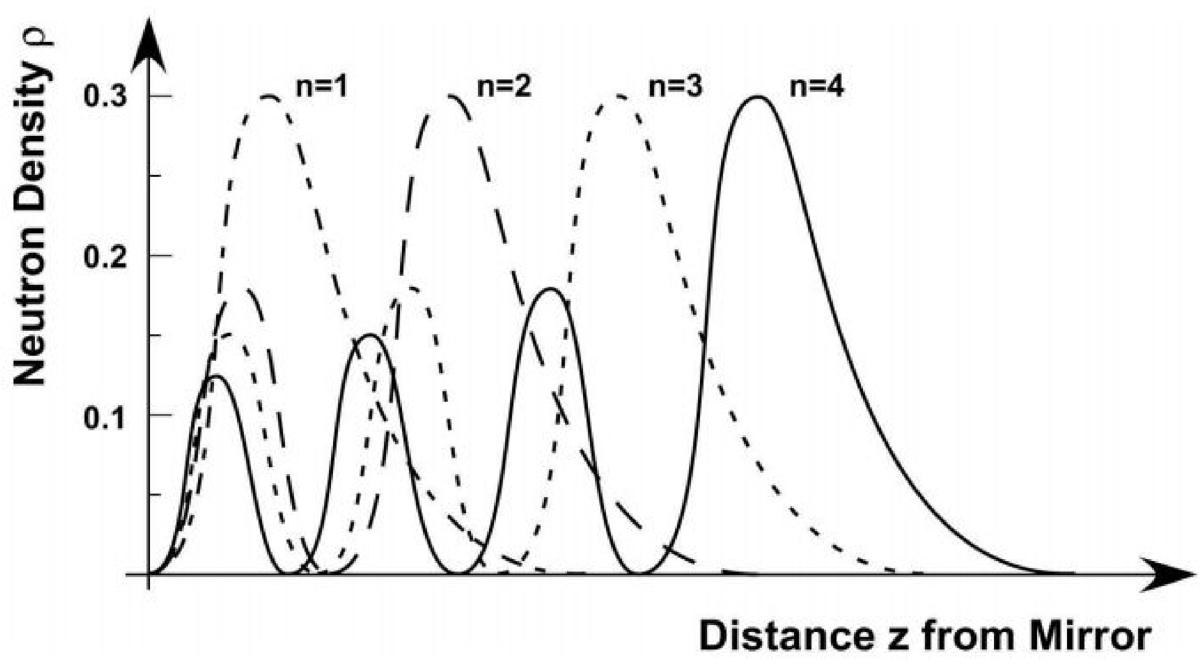

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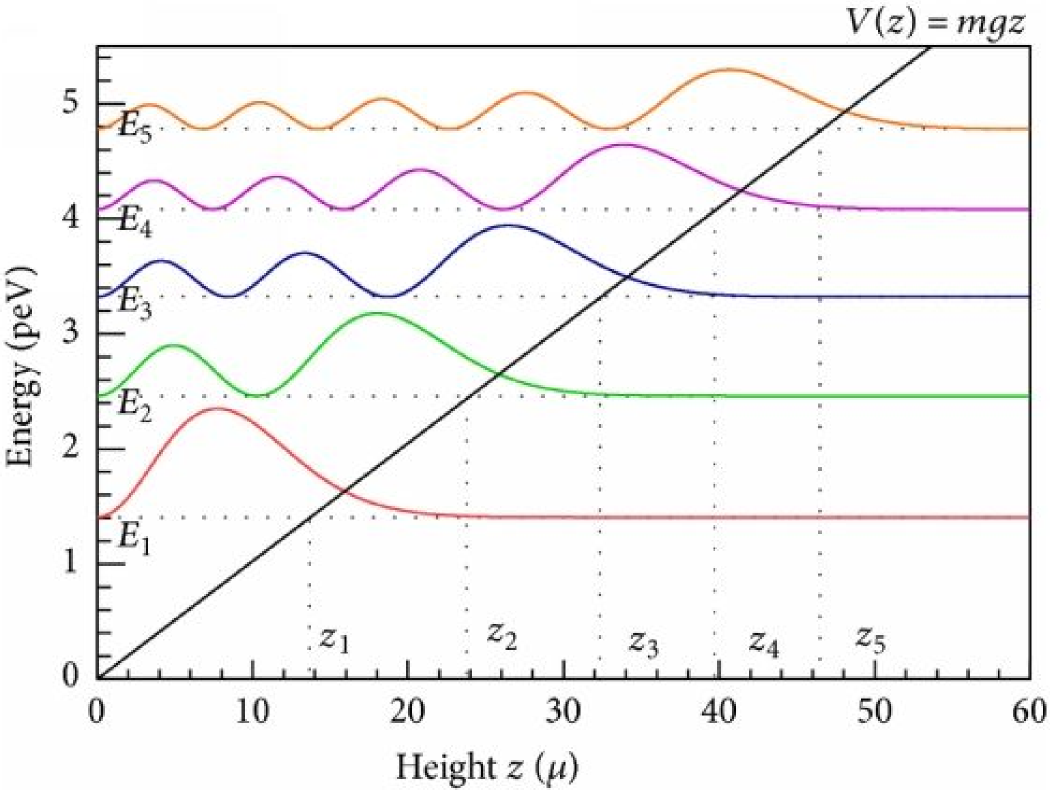
Figure 5.



Y. Kamiya, G. Ichikawa, and S. Komamiya, *Precision Measurement of the Position-Space Wave Functions of Gravitationally Bound Ultracold Neutrons*, Advances in High Energy Physics Volume 2014 (2014), Article ID 859241

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